5-10 Factoring by Grouping

Objective: To factor a polynomial by grouping terms.

Example 1   Factor:
   a. $3(x - y) + w(x - y)$
   b. $m(m + 3n) - (m + 3n)$
   c. $r(p - q) + s(p - q) + t(p - q)$

Solution   Use the distributive property: $ba + ca = (b + c)a$.

This property is valid when a represents any polynomial. For example:

If $a = x - y$, you have $b(x - y) + c(x - y) = (b + c)(x - y)$.

   a. $3(x - y) + w(x - y) = (3 + w)(x - y)$

   b. $m(m + 3n) - (m + 3n) = m(m + 3n) - 1(m + 3n)$
   $$= (m - 1)(m + 3n)$$

   c. $r(p - q) + s(p - q) + t(p - q) = (r + s + t)(p - q)$

Factor.

1. $2(x + y) + z(x + y)$

2. $5(a - b) + c(a - b)$

3. $e(f + g) - 4(f + g)$

4. $w(x - y) - 6(x - y)$

5. $(c + 2d) - e(c + 2d)$

6. $2c(a - b) - (a - b)$

7. $2x(m - n) - (m - n)$

8. $r(p - q) - (p - q)$

9. $3u(u - 2v) + v(u - 2v) + (u - 2v)$

10. $c(a + b) - d(a + b) + e(a + b)$

Example 2   Factor $7(a - 2) - a(2 - a)$.

Solution   Notice that $a - 2$ and $2 - a$ are opposites.

$7(a - 2) - a(2 - a) = 7(a - 2) - a[-(a - 2)]$ Write $-(a - 2)$ for $2 - a$.
$$= 7(a - 2) + a(a - 2)$$ Use the distributive property.
$$= (7 + a)(a - 2)$$

Check: $(7 + a)(a - 2) = 7a - 14 + a^2 - 2a$
$$= (7a - 14) + (a^2 - 2a)$$
$$= 7(a - 2) - (2a - a^2)$$
$$= 7(a - 2) - a(2 - a) \; \checkmark$$

Therefore, $7(a - 2) - a(2 - a) = (7 + a)(a - 2)$.

Factor. Check by multiplying the factors.

11. $2x(m - n) - (n - m)$

12. $w(x - y) - 7(y - x)$

13. $6(r - s) + t(s - r)$

14. $6(m - n) + p(n - m)$

15. $u(v - 3) + 3(3 - v)$

16. $3x(x - y) + y(y - x)$

17. $x(x - 5) - (5 - x)$

18. $h(h - 6) - 2(6 - h)$

5–10 Factoring by Grouping (continued)

Example 3   Factor $ax - 2x + ay - 2y$.

Solution 1 $\begin{aligned} ax - 2x + ay - 2y &= (ax - 2x) + (ay - 2y) \\ &= x(a - 2) + y(a - 2) \\ &\doteq (x + y)(a - 2) \end{aligned}$ Group terms with common factors.
Factor each group of terms.
Use the distributive property.

Solution 2 $\begin{aligned} ax - 2x + ay - 2y &= (ax + ay) - (2x + 2y) \\ &= a(x + y) - 2(x + y) \\ &= (a - 2)(x + y) \end{aligned}$ Group terms with common factors.
Factor each group of terms.
Use the distributive property.

Factor. Check by multiplying the factors.

19. $2a + ab + 2c + bc$ **20.** $rs - 6r + st - 6t$

21. $x^2 - 3x + xy - 3y$ **22.** $u^2 + 3u + uv + 3v$

23. $xy - xz - 3y + 3z$ **24.** $5t - 10 - st + 2s$

25. $mx + m + 3x + 3$ **26.** $5x - 5y + wx - wy$

27. $5m^3 - 3m^2 + 10m - 6$ **28.** $2a^3 + a^2 - 6a - 3$

29. $a^2 - 3ab + ac - 3bc$ **30.** $2ab - b - 4a + 2$

31. $2u^3 - u^2 - 4u + 2$ **32.** $x^3 - 4x^2 - x + 4$

Example 4   Factor $(a + 2b)^2 - c^2$ as a difference of two squares.

Solution $\begin{aligned} (a + 2b)^2 - c^2 &= [(a + 2b) + c][(a + 2b) - c] \\ &= (a + 2b + c)(a + 2b - c) \end{aligned}$ $\left\{ \begin{aligned} &\text{Use the pattern} \\ &a^2 - b^2 = (a + b)(a - b). \end{aligned} \right.$

Factor as a difference of squares.

33. $(a - b)^2 - 4c^2$ **34.** $(x + 3y)^2 - 16z^2$

35. $x^2 - (y + z)^2$ **36.** $9p^2 - (q - 2r)^2$

37. $m^2 - (n + 3)^2$ **38.** $h^2 - (k - 6)^2$

39. $m^2 - (n - 1)^2$ **40.** $4(x - y)^2 - 25$

Mixed Review Exercises

Solve.

1. $-10 + x = -27$ **2.** $-n + 8 = 3$

3. $16 + x = 34$ **4.** $13 = 1 + 3x$

5. $9m - 6m = 27$ **6.** $4n - 2n + 6 = 12$

7. $12x = 600$ **8.** $-11m = 143$

9. $7b = 105$ **10.** $9n = 3n - 30$

11. $17m = 44 + 13m$ **12.** $9y + 3 = 3(17 - y)$